Received: *February 2, 2001,* Accepted: *March 13, 2001*

Geometry on string lattice

Naoki Sasakura

Department of Physics, Stanford University
Stanford, CA 94305-4060, USA, and
Yukawa Institute for Theoretical Physics, Kyoto University
*Kyoto 606-8502, Japan**
E-mail: naokisa@stanford.edu, sasakura@yukawa.kyoto-u.ac.jp

Abstract: Using the method developed by Callan and Thorlacius, we study the low energy effective geometry on a two-dimensional string lattice by examining the energy-momentum relations of the low energy propagation modes on the lattice. We show that the geometry is identical for both the oscillation modes tangent and transverse to the network plane. We determine the relation between the geometry and the lattice variables. The lowest order effective field theory is given by the dimensional reduction of the ten-dimensional $N = 1$ Maxwell theory. The gauge symmetry is related to a property of a three-string junction but not of a higher order junction. A half of the supersymmetries in the effective field theory should be broken at high energy.

Keywords: Lattice Quantum Field Theory, Superstrings and Heterotic Strings, D-branes, Brane Dynamics in Gauge Theories.

*Permanent address

JHEP03(2001)017

Contents

1. Introduction

String junction [1, 2] in type-IIB string theory is known as a BPS object preserving a quarter of the supersymmetries of type-IIB string theory [3, 4]. Connecting many of these junctions, string network can be constructed [4]. When the whole network is stabilized within a two-dimensional plane, the network as a whole was also shown to be a quarter BPS state [4]. Protected by these remaining supersymmetries, there exist rigid zero modes of string network [5, 6]. They are roughly the modes changing the sizes of each loop and can even reconnect the strings to change the topology of a network. Since these zero modes exist locally in a string network, it is generally an object which can have a macroscopic entropy [7]. In this paper, we will continue pursuing the possibility whether a string lattice can be regarded as a kind of continuous space in the low energy limit [8], like in lattice gauge theory. This way of view might be similar in spirit to the idea of brane world [9, 10] and also to the network appearing in quantum gravity [11]–[15].

A main question in regarding a string lattice as a continuous space is what is the low energy dynamics on it. The dynamics of a string network was analyzed in the small oscillation limit in [16, 17], and a general framework for studying the moduli space of the above mentioned zero modes was recently given in [18]. The question of what kind of field theory is on a string lattice was studied in our previous paper [8] for the case of a regular hexagonal lattice. There the oscillations of a string lattice is approximated by the oscillations of junctions between which there are straight strings. This approximation of straight strings would be valid when the motions of the string junctions are much slower than the motions of the oscillations on the strings. Our main interest was that, since the zero modes exist locally in the network, they may appear as a local gauge symmetry in the low energy effective field theory. In fact, we have obtained the result that the oscillation mode tangent to the network plane is described by Maxwell theory, the gauge symmetry of which comes from these zero modes. As for the oscillations transverse to the network plane, we have obtained a scalar field theory with seven scalars.

The low energy supersymmetries of the effective field theory give a good relation between the Maxwell and the scalar modes. From the supergravity argument of [4], the eight supersymmetries remaining on the network are in the spinor representation **8** of the SO(7) symmetry which is the rotation among the directions transverse to the network. In [17], the eight supersymmetries were identified from the world sheet view point. There are one supersymmetry for each of the seven transverse directions and another for the mode along the network. Thus the remaining supersymmetries and the SO(7) symmetry should mix up the scalar fields and the Maxwell field.

The approximate treatment in our previous paper [8] can be well controlled by assigning a point-like mass to each junction. If we take the mass sufficiently large, the oscillations of the junctions will become sufficiently slower than the string oscillations, which will validate our approximation. However, in the limit of vanishing mass, which is the case we are interested in, we cannot fully believe the validity of our approximation.

In this paper we will instead use the method developed in [17] to investigate the low energy dynamics of a periodic string lattice, and will generalize our previous results [8]. In section 2, we will recapitulate and discuss some properties of the scattering matrix associated to junctions and the low energy oscillation modes on a string lattice. We will show that the gauge symmetry is a general feature of a string lattice made of three-string junctions, but not of one made of higher order junctions. In section 3, we will extract the low energy geometry by studying the energy-momentum relation of the low energy propagation modes. The geometry is identical for both the tangent and transverse modes, so that one geometry can be assigned to a string lattice. It is also shown to be consistent with the gauge symmetry. In section 4, we will discuss the low energy effective field theory and its supersymmetry. Section 5 is devoted for summary and discussions.

2. The gauge symmetry

In this paper we will use the approach to the dynamics of the small oscillations of string junction which was developed in [17]. Their approach is to relate the in-wave and out-wave at a string junction by a matrix to characterize the dynamics of the string junction. In their small oscillation limit, the dynamics is independent for the mode in each of the directions transverse to the junction plane (out-plane) and the mode along the junction plane (in-plane). Let us consider the fluctuations $\phi_i(x_i,t)$ on a string in one of those independent directions, where i $(i=1,\ldots,N)$ is the index for the strings forming an N-order string junction. The oscillation on a string can be written as

$$\phi_i(x_i,t) = \mathrm{Re}\{(A_i \exp(i\omega x_i) + B_i \exp(-i\omega x_i)) \exp(-i\omega t)\}, \tag{2.1}$$

where A_i, B_i are the complex mode amplitudes of out-wave and in-wave, respectively, and $x_i > 0$ is the distance from the junction measured along the i-th string. The oscillations slightly deform the configuration of the strings near the string junction so that they satisfy the following two physical matching conditions at the junction. One is the continuity condition that the strings should meet at one point, and the other is the tension balance condition that the forces from the string tensions pulling the junction should add up to vanish. From these conditions they determined the matrix relating the in-wave and out-wave amplitudes, $\vec{A} = S\vec{B}$, for a general N-order string junction. In the case of a three-string junction ($N = 3$), the matrix S is given by

$$S_{\text{out-plane}} = -\mathbf{1} + \frac{2}{\sum_{i=1}^{3} t_i} \begin{pmatrix} t_1 & t_2 & t_3 \\ t_1 & t_2 & t_3 \\ t_1 & t_2 & t_3 \end{pmatrix} \tag{2.2}$$

for an out-plane scattering, and, for an in-plane scattering,

$$S_{\text{in-plane}} = \mathbf{1} - \frac{2}{D} \begin{pmatrix} t_2t_3 \sin^2\theta_{23} & t_2t_3 \sin\theta_{23}\sin\theta_{31} & t_2t_3 \sin\theta_{23}\sin\theta_{12} \\ t_1t_3 \sin\theta_{31}\sin\theta_{23} & t_1t_3 \sin^2\theta_{31} & t_1t_3 \sin\theta_{31}\sin\theta_{12} \\ t_1t_2 \sin\theta_{12}\sin\theta_{23} & t_1t_2 \sin\theta_{12}\sin\theta_{31} & t_1t_2 \sin^2\theta_{12} \end{pmatrix}, \tag{2.3}$$

where t_i's and θ_{ij}'s are the string tensions and the angles between the string i and j, respectively, and $D = t_1t_2\sin^2\theta_{12} + t_2t_3\sin^2\theta_{23} + t_3t_1\sin^2\theta_{31}$.

A fairly convenient coordinate can be taken for these matrices [17]. After a similarity transformation, $\hat{S} = \sqrt{t}S\sqrt{t}^{-1}$, where t is a diagonal matrix whose entries are the tensions, they found

$$\hat{S}_{\text{out-plane}} = -\mathbf{1} + 2\vec{y}\otimes\vec{y}\,, \tag{2.4}$$

where $\vec{y}$ is a unit vector

$$\vec{y} = \frac{1}{\sqrt{\sum t_i}}(\sqrt{t_1}, \sqrt{t_2}, \sqrt{t_3})\,, \tag{2.5}$$

and

$$\hat{S}_{\text{in-plane}} = \mathbf{1} - 2\vec{z}\otimes\vec{z}\,, \tag{2.6}$$

where $\vec{z}$ is a unit vector

$$\vec{z} = \frac{1}{\sqrt{D}}(\sqrt{\tau_{23}}, \sqrt{\tau_{31}}, \sqrt{\tau_{12}}) \tag{2.7}$$

with $\tau_{ij} = t_it_j\sin^2\theta_{ij}$. The eigenvalues of these matrices are $(1,-1,-1)$ and $(1,1,-1)$, respectively, and the eigenvalues 1 and -1 correspond to Neumann and Dirichlet boundary conditions generalized to a junction formed by strings with arbitrary tensions, respectively.

Although they did not mention explicitly in their paper, the vectors $\vec{y}$ and $\vec{z}$ are the same. This can be shown from the relations among the angles θ_{ij} and the tensions t_i. Due to the tension balance condition, the angles $\pi - \theta_{ij}$ and the tensions t_i are in fact the angles and the edge lengths of a closed triangle, respectively, and so they should satisfy

$$\frac{t_1}{\sin\theta_{23}} = \frac{t_2}{\sin\theta_{31}} = \frac{t_3}{\sin\theta_{12}} . \tag{2.8}$$

From these equations we obtain

$$(\tau_{23}, \tau_{31}, \tau_{12}) = C(t_1, t_2, t_3) , \tag{2.9}$$

where C is an unimportant positive factor. Thus we obtain

$$\vec{y} = \vec{z} . \tag{2.10}$$

This gives a simple relation between the out-plane and in-plane scattering matrices:

$$S_{\text{out-plane}} = -S_{\text{in-plane}} . \tag{2.11}$$

As we will see, because of this relation, the spectra of the oscillation modes on a periodic string lattice in the tangent and transverse directions are basically the same, and this is consistent with the discussions about the remaining supersymmetries in section 1.

As for a higher order string junction, the eigenvalues of the scattering matrices are $(1, -1, \ldots, -1)$ and $(1, 1, -1, \ldots, -1)$ for out-plane and in-plane, respectively [17]. Hence we can not expect a simple relation like (2.11) to hold between the two kinds of scattering matrices in this case. Thus, when a string lattice is made of higher order junctions, it is obscure how the low energy field theory respects the eight remaining supersymmetries expected from the supergravity argument.

The spectra of the oscillations on an infinite periodic string lattice can be analyzed by using the scattering matrices. The case that a string lattice is given by a periodic connection of one kind of three-string junction was analyzed in [17]. The basic equations are given by

$$\begin{aligned} \vec{A} &= S\,\vec{B}, \\ P\,\alpha\,\vec{B} &= S\,P^*\,\alpha\,\vec{A} , \end{aligned} \tag{2.12}$$

where S is $S_{\text{out-plane}}$ or $S_{\text{in-plane}}$ for the out-plane or in-plane modes, respectively. Here $P = \mathrm{diag}(\exp(-i\omega l_1), \exp(-i\omega l_2), \exp(-i\omega l_3))$ is a diagonal matrix accounting for the phase shifts at the other ends of each string with length l_i forming a three-string junction, and $\alpha = \mathrm{diag}(\exp(i\alpha_1), \exp(i\alpha_2), \exp(i\alpha_3))$ is a diagonal matrix of the phase shifts associated to the discrete translation symmetries of the periodic lattice. The momenta of the oscillation modes are related to α by

$$\alpha_i = \vec{l}_i \cdot (p_1, p_2) , \tag{2.13}$$

where $\vec{l}_i$ denote the vectors in the two-dimensional lattice plane with the directions and lengths of the strings starting from the junction point, and $\cdot$ is the inner product between vectors on the plane. The mode spectra, which give relations between ω and (p_1, p_2), can be determined by analyzing the condition that the equations (2.12) have nontrivial solutions for $\vec{A}, \vec{B}$. Because of (2.11), the problem is basically equivalent for the out-plane and in-plane cases.

As was noticed in [17], there is a special solution to the equations (2.12). This solution exists for any momenta (p_1, p_2) without any cost of energy: $\omega = 0$. These modes were identified as the gauge symmetry of the low energy effective action in our previous paper [8]. The reason for this identification is that this fact shows that these modes are the time-independent local deformations of the string lattice configuration under which the potential energy remains unchanged. Let us look into the modes in more detail by solving (2.12) for $P = \mathbf{1}$ $(\omega = 0)$. Since α is commutative with the tension matrix t, the problem of solving (2.12) is easier after the similarity transformation by $\sqrt{t}$. Then, by eliminating $\vec{B}$, we obtain

$$\vec{\hat{A}} = \hat{S}_{\text{in-plane}} \hat{S}_\alpha \vec{\hat{A}}\,, \tag{2.14}$$

where

$$\hat{S}_\alpha = 1 - 2(\alpha \vec{y})^* \otimes (\alpha \vec{y})\,. \tag{2.15}$$

Thus the solution to (2.14) for $\vec{\hat{A}}$ is given explicitly by a vector orthogonal both to $\vec{y}$ and $\alpha\vec{y}$:

$$\vec{\hat{A}}_{\text{gauge}} = (\alpha \vec{y}) \times \vec{y}\,, \tag{2.16}$$

where $\times$ denotes an outer product in the three-dimensional string index space. As we mentioned, the equations for the oscillation spectra are basically equivalent for the out-plane and in-plane, but the physical spectra are different. From (2.1), for $\omega = 0$, the physical quantities are $(\vec{A} + \vec{B})_{\text{out-plane}} = (1 - S_{\text{in-plane}})\vec{A}$ and $(\vec{A} + \vec{B})_{\text{in-plane}} = (1 + S_{\text{in-plane}})\vec{A}$ for the out-plane and in-plane, respectively. Since the solution satisfies $\hat{S}_{\text{in-plane}} \vec{\hat{A}}_{\text{gauge}} = \vec{\hat{A}}_{\text{gauge}}$, the gauge mode $\vec{\hat{A}}_{\text{gauge}}$ does not exist as an out-plane mode, while it does as an in-plane mode. This makes a difference between the low energy effective field theories for the oscillations transverse and tangent to the lattice plane.

As for a higher order junction $(N \geq 4)$, the existence of the gauge mode is not expected. This can be shown as follows. From the analysis of [17], the rescaled matrix of the in-plane scattering matrix for an N-string junction, $\hat{S}_{\text{in-plane}} = \sqrt{t} S_{\text{in-plane}} \sqrt{t}^{-1}$, should have the from

$$\hat{S}_{\text{in-plane}} = \mathbf{1} - 2 \sum_{i=1}^{N-2} \vec{z}_i \otimes \vec{z}_i\,, \tag{2.17}$$

where $\vec{z}_i$'s are orthogonal unit vectors. By a similar argument as above, in this N-junction case,

$$\hat{S}_\alpha = \mathbf{1} - 2\sum_{i=1}^{N-2}(\alpha\vec{z}_i)^* \otimes (\alpha\vec{z}_i)\,. \tag{2.18}$$

Since there does not seem to exist any necessity for the vectors $\{\vec{z}_i, \alpha\vec{z}_i\}$ to degenerate to less than N-dimensions, we do not expect that there exists a vector orthogonal to all $\{\vec{z}_i, \alpha\vec{z}_i\}$ for $N \geq 4$. This fact can be explicitly checked for some simple examples. This is consistent with the argument that the gauge symmetry comes from the zero modes of changing the sizes of each loop of a network [8]. We cannot change the size of a loop without changing the sizes of the other neighboring loops when higher-order junctions are forming the loop. Thus we do not obtain local zero modes in such a case.

3. Geometry on string lattice

As for the interest in the possibility to regard a string lattice as a continuous space manifold in the low energy limit, it would be interesting to investigate the effective low energy geometry on a string lattice. This will be extracted from the relations between the energy and momentum of the low-energy propagation modes, $\omega^2 = \sum_{m,n=1}^{2} g^{mn} p_m p_n$.

The condition for the equations (2.12) to have non-trivial solutions was obtained in [17]:

$$\begin{aligned} 0 = &\left(\sum_{i=1}^{3} t_i^2\right) s_1 s_2 s_3 + 2t_1 t_2 s_3(\cos(\alpha_1-\alpha_2) - c_1 c_2) + \\ &+ 2t_2 t_3 s_1(\cos(\alpha_2-\alpha_3) - c_2 c_3) + 2t_3 t_1 s_2(\cos(\alpha_3-\alpha_1) - c_3 c_1), \end{aligned} \tag{3.1}$$

where $s_i = \sin(\omega l_i), c_i = \cos(\omega l_i)$. As we have discussed in the previous section, there is a rather trivial solution with $\omega = 0$ for any given (p_1, p_2). In addition, there exists another low energy solution in (3.1). This is the low energy propagation mode which we are interested in in this section. Expanding (3.1) in the cubic order of ω and the quadratic order of α_i, we obtain

$$\omega^2 = \frac{1}{L}\left(t_1 t_2 l_3(\alpha_1-\alpha_2)^2 + t_2 t_3 l_1(\alpha_2-\alpha_3)^2 + t_3 t_1 l_2(\alpha_3-\alpha_1)^2\right), \tag{3.2}$$

where

$$L = (t_1 l_2 l_3 + t_2 l_3 l_1 + t_3 l_1 l_2)(l_1 t_1 + l_2 t_2 + l_3 t_3)\,. \tag{3.3}$$

Thus, substituting this with (2.13), the two-dimensional metric tensor is obtained as

$$g^{mn} = \frac{1}{L}\sum_{\substack{i<j,\\ k\neq i,j}}^{3} t_i t_j l_k (\vec{l}_i - \vec{l}_j)^m (\vec{l}_i - \vec{l}_j)^n\,. \tag{3.4}$$

A more convenient expression of the metric tensor can be obtained as follows. Let us pick up the terms with the form $\vec{l}_1 \otimes \cdots$ in the numerator of the metric tensor (3.4):

$$\vec{l}_1 \otimes \Big(t_1 t_2 l_3 (\vec{l}_1 - \vec{l}_2) - t_3 t_1 l_2 (\vec{l}_3 - \vec{l}_1) \Big) \,. \tag{3.5}$$

Let us define the tension vectors in the two-dimensional lattice plane by $\vec{t}_i = (t_i/l_i)\vec{l}_i$ (no sum). From the tension balance condition, these vectors satisfy

$$\sum_{i=1}^{3} \vec{t}_i = 0 \,. \tag{3.6}$$

Substituting $\vec{l}_i$ with $\vec{t}_i$ in (3.5) and using (3.6), we obtain

$$(t_1 l_2 l_3 + t_2 l_3 l_1 + t_3 l_1 l_2)\ \vec{l}_1 \otimes \vec{t}_1 \,. \tag{3.7}$$

Thus another equivalent expression for the metric tensor is given by

$$g^{mn} = \frac{\sum_{i=1}^{3} (\vec{l}_i)^m (\vec{t}_i)^n}{\sum_{i=1}^{3} l_i t_i} \,. \tag{3.8}$$

This expression clearly shows that, for any momentum (p_1, p_2), the inequality $\omega^2 = g^{mn} p_m p_n < \sum_{n=1}^{2} p_n p_n$ holds. This means that the velocity of the low energy propagation mode is always smaller than the light velocity for an observer in the target space.

Now let us study the polarization of the gauge symmetry obtained in the previous section. Since the definition of the momentum (2.13) is through the inner product with vectors in the target space, the momentum has a contravariant index, while the polarization of the gauge symmetry has a covariant index since the field ϕ of (2.1) denotes string motions in the target space. Thus we need the metric tensor to see the relation between the momentum and the polarization. We will show below that the polarization is certainly parallel to the momentum when we take into account the metric (3.8) (or (3.4)).

From the result (2.16), in the first order of the momentum, the in-wave amplitude of the gauge symmetry is given by

$$\begin{aligned}(A_1, A_2, A_3)_{\rm gauge} &= \sqrt{t}^{\,-1} (\hat{A}_1, \hat{A}_2, \hat{A}_3)_{\rm gauge} \\ &= \left(\sqrt{\frac{t_2 t_3}{t_1}} (\vec{l}_2 - \vec{l}_3) \cdot \vec{p}, \sqrt{\frac{t_3 t_1}{t_2}} (\vec{l}_3 - \vec{l}_1) \cdot \vec{p}, \sqrt{\frac{t_1 t_2}{t_3}} (\vec{l}_1 - \vec{l}_2) \cdot \vec{p} \right)\end{aligned} \tag{3.9}$$

up to an unimportant all over factor. Since $\vec{A} + \vec{B} = 2\vec{A}$ in this case, (3.9) represents the physical polarization. To extract the two-dimensional in-plane polarization vector from the data (3.9), we will proceed as follows. Let us denote the corresponding two-dimensional polarization vector by $\vec{\varphi}_{\rm gauge}$, and suppose it is expanded by the tension vectors: $\vec{\varphi}_{\text{gauge}} = \sum_{i=1}^{3} \varphi_i \vec{t}_i$.[1] Then, since A_i denote the fluctuations transverse to each string i, we have

$$\begin{aligned} A_i &= \vec{\varphi}_{\text{gauge}} \cdot \frac{R(\vec{t}_i)}{t_i} \\ &= \sum_{j=1}^{3} \varphi_j \vec{t}_j \cdot \frac{R(\vec{t}_i)}{t_i} \\ &= \sum_{j=1}^{3} \varphi_j t_j \sin(\theta_{ij}) \,, \end{aligned} \tag{3.10}$$

where $R(\cdot)$ denotes a two-dimensional rotation operator of degree $\pi/2$. Thus, using (2.8) and setting $T = t_1/\sin(\theta_{23}) = \cdots$, we obtain

$$(t_1 A_1, t_2 A_2, t_3 A_3) = \frac{t_1 t_2 t_3}{T}\,(1,1,1) \times (\varphi_1, \varphi_2, \varphi_3)\,. \tag{3.11}$$

Comparing with the data (3.9), we obtain

$$(\varphi_1, \varphi_2, \varphi_3) = \frac{T}{\sqrt{t_1 t_2 t_3}} (\vec{l}_1 \cdot \vec{p}, \vec{l}_2 \cdot \vec{p}, \vec{l}_3 \cdot \vec{p})\,. \tag{3.12}$$

Thus we finally obtain the desired result

$$\begin{aligned} (\vec{\varphi}_{\text{gauge}})^m &= \frac{T}{\sqrt{t_1 t_2 t_3}} \sum_{i=1}^{3} (\vec{t}_i)^m (\vec{l}_i \cdot \vec{p}) \\ &= C g^{mn} p_n \,, \end{aligned} \tag{3.13}$$

where C is an unimportant factor.

4. Effective field theory and supersymmetry

Let us first discuss the effective action for one of the oscillations transverse to the network plane. In the small oscillation limit, the kinetic term of the effective action should be well approximated by

$$K = \int dt\, \frac{m}{2} \sum_{j=\text{junction}} \left(\frac{d\phi_j}{dt}\right)^2 , \tag{4.1}$$

where $m = (l_1 t_1 + l_2 t_2 + l_3 t_3)/2$ denotes the mass per junction, and ϕ_j is the oscillation of a junction j. The action per junction may be smeared on the two-dimensional lattice plane, and we may introduce a coordinate (z_1, z_2), which is just the target

[1] Since $\sum_{i=1}^{3} \vec{t}_i = 0$, this is a degenerate expression. In general this expansion will not work well, but for our present purpose, this coordinate will turn out to be very convenient.

JHEP03(2001)017

space coordinate restricted on the lattice plane, to parameterize the collective field for the transverse oscillation. Then (4.1) will be rewritten in the form

$$K_{\rm col} = \int dt d^2 z \, \frac{m}{2s} \left(\frac{d\phi_{\rm col}}{dt} \right)^2 , \tag{4.2}$$

where s is the area per vertex, and is explicitly given by

$$\begin{aligned} s &= \frac{\sqrt{q}}{4t_1t_2t_3}(l_1l_2t_3 + l_2l_3t_1 + l_3l_1t_2), \\ q &= (t_1+t_2+t_3)(t_1+t_2-t_3)(t_2+t_3-t_1)(t_3+t_1-t_2) \, . \end{aligned} \tag{4.3}$$

On the other hand, after a straightforward calculation, the determinant of the metric tensor (3.8) is given by

$$g^{-1} = {\rm Det}(g^{mn}) = \frac{(l_1l_2t_3 + l_2l_3t_1 + l_3l_1t_2)q}{4t_1t_2t_3(t_1l_1 + t_2l_2 + t_3l_3)^2} \, . \tag{4.4}$$

Hence the action for the collective field (4.2) is

$$K_{\rm col} = \frac{1}{2} \int dt d^2 z \, \sqrt{g} \left(\frac{t_1t_2t_3}{l_1l_2t_3 + l_2l_3t_1 + l_3l_1t_2} \right)^{\frac{1}{2}} \left(\frac{d\phi_{\rm col}}{dt} \right)^2 . \tag{4.5}$$

We do not have any explanations for the extra factor in (4.5). Anyway, in our present approximation without any interactions and a coordinate dependence of the lattice variables t_i, l_i, we may freely rescale the fields. Since the collective field $\phi_{\rm col}$ has mass dimension -1, it is natural to rescale this field by a factor with a non-zero mass dimensions and define a new field with the canonical dimension $1/2$:

$$\phi = \left(\frac{t_1t_2t_3}{l_1l_2t_3 + l_2l_3t_1 + l_3l_1t_2} \right)^{1/4} \phi_{\rm col} \, . \tag{4.6}$$

The potential term can be determined from the condition that the energy-momentum relation be generated correctly. Thus the effective action of the transverse modes should be

$$S_{\rm transverse} = - \int d^3 x \sqrt{-\det(g_{\mu\nu})} \sum_{i=1}^{7} \left(g^{\mu\nu} \frac{\partial \phi^i}{\partial x^\mu} \frac{\partial \phi^i}{\partial x^\nu} \right) , \tag{4.7}$$

where

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & g^{mn} \end{pmatrix} \tag{4.8}$$

and $x^\mu = (t, z^i)$.

As for the tangent modes, the collective field is given by a two-dimensional vector field. Following the same discussions given in our previous paper [8], the potential term of the low energy Lagrangian has the gauge symmetry studied in the previous sections, and hence the effective theory is expected to be Maxwell theory. The action is

$$S_{\rm tangent} = -\int d^3x \sqrt{-\det(g_{\mu\nu})}\frac{1}{4}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \,. \tag{4.9}$$

The kinetic term of this action takes the form

$$\frac{1}{2}g_{mn}\frac{\partial a^m}{\partial t}\frac{\partial a^n}{\partial t}\,, \tag{4.10}$$

where a^m $(m = 1, 2)$ is the gauge potential, which represents the collective in-plane motion of the string lattice in the target space-time. This kinetic term cannot be derived from a similar simple argument as above for the scalar fields that an average mass m is assigned to each junction. This fact seems to indicate a subtlety of the in-plane string dynamics, and another more careful treatment is necessary.

Since the metric $g^{\mu\nu}$ is just constant in our periodic string lattice, from now on, we change the coordinate and work in a coordinate with Minkowski metric $g^{\mu\nu} = \mathrm{diag}(-1, 1, 1)$ in the following discussions about the fermionic sector and supersymmetries. The spectra of the fermionic sector on a string lattice have also been discussed in [17]. There they showed that the remaining world-sheet supersymmetries on strings are consistent with the boundary conditions at junctions, so that there is a one-to-one correspondence between the bosonic and fermionic spectra. Thus there exist one fermionic low energy propagation mode satisfying $\omega = \pm\sqrt{g^{mn}p_m p_n}$ per each of the transverse directions and one in-plane fermionic mode. The simplest fermion in $(2+1)$-dimensions is a Majorana fermion, which has two real component. The Dirac equation for a Majorana fermion generates just what we want as the spectra for the low energy fermionic modes on a string lattice. Thus the action for the fermion should be

$$S_{\rm fermion} = \int d^3x \sum_{\beta=1}^{8} \psi_\beta^T \Gamma^0 \Gamma^\mu \partial_\mu \psi_\beta \,, \tag{4.11}$$

where we may take any two-by-two real representation of Γ^μ.

On the other hand, Sen discussed the remaining supersymmetries by a supergravity argument [4]. The remaining supersymmetries are the solutions of

$$\begin{aligned}
\epsilon_L &= \Gamma^0_{(10)}\Gamma^1_{(10)}\epsilon_L \,, \\
\epsilon_R &= -\Gamma^0_{(10)}\Gamma^1_{(10)}\epsilon_R \,, \\
\epsilon_L &= \Gamma^0_{(10)}\Gamma^2_{(10)}\ \epsilon_R \,,
\end{aligned} \tag{4.12}$$

where $\Gamma^\mu_{(10)}$ are the (9+1)-dimensional gamma matrices, and ϵ_L and ϵ_R are the Majorana-Weyl spinor **16** of $SO(9,1)$. Studying (4.12), there remain eight supersymmetries, and the solutions to the first equation of (4.12) can be taken as the independent components. In our case, the ten-dimensional space-time is divided into the (2+1)-dimensional lattice space-time and its transverse seven-dimensional space. Since **16** of $SO(9,1)$ is $\mathbf{2} \times \mathbf{8}$ of $SO(2,1) \times SO(7)$ and the first equation of

(4.12) gives a condition just for the SO(2, 1) part,[2] the remaining supersymmetries are in the spinor representation **8** of SO(7). On the other hand the gauge field and the scalar fields in the effective action is in the singlet and vector representation of SO(7), respectively. To combine the bosonic and fermionic fields into a multiplet the fermions in the effective action should be in the spinor representation. This pattern of the multiplet is just what we will obtain from the dimensional reduction of the $(9+1)$-dimensional $N = 1$ vector multiplet to $(2+1)$-dimensions. Thus we conclude that the low energy effective theory is just given by the dimensional reduction of the $N = 1$ $(9+1)$-dimensional super Maxwell theory.

This result sounds rather curious, since the number of the supersymmetries of the effective action should be eight rather than sixteen. In the low-energy effective action there is the two-dimensional rotational symmetry, and so there are no good ways to impose non-rotationally symmetric conditions as those in (4.12) on the sixteen supersymmetries of the effective action. On the other hand at sufficiently high energy, the bare lattice structure will contribute to the dynamics. Thus it is expected that, although the supersymmetries are double in the effective action within our range of approximation of this paper, the higher order terms relevant at sufficiently high energy are expected to break a half of them.

5. Summary and discussions

In this paper, we have discussed the low energy effective geometry on a two-dimensional string lattice, by studying the energy-momentum relations of the low energy propagation modes. An interesting result is that the geometry is the same for the tangent and transverse oscillations, which makes it possible to assign one geometry to a given string lattice.

We have also discussed the effective action of the low energy propagation modes. We have obtained the dimensional reduction of $N = 1$ $(9+1)$-dimensional supersymmetric Maxwell theory. A half of the sixteen supersymmetries are expected to be broken at high energy, where higher order terms out of our range of approximation become relevant.

We should be careful about the fact that all the results in this paper are based on the assumption that the oscillations of a string lattice are small. This does not seem to be a justifiable assumption because the zero modes may considerably change the local structure of the lattice and even its topology.[3] As one can see from the explicit expression of the geometry (3.8), the changes of the string lengths change

[2]This SO(2, 1) symmetry should not be confused with the same symmetry of the effective action, because the propagation velocity of the low energy modes is different from the light velocity of the target space.

[3]This topology change in type-IIB string theory can be viewed in a different interesting way when a string network is lifted to M-theory. See [18] for the recent discussions.

JHEP03(2001)017

the low energy effective geometry on a string lattice, and so the zero mode dynamics might be related to the gravity side of the effective field theory. This argument is yet very uncertain, and it seems an important open matter to identify the roles of these zero modes in the low energy effective action.

Acknowledgments

The author is supported in part by the Fellowship Program for Japanese Scholars and Researchers to study abroad, in part by Grant-in-Aid for Scientific Research (#12740150), and in part by Priority Area: "Supersymmetry and Unified Theory of Elementary Particles" (#707), from Ministry of Education, Science, Sports and Culture, Japan.

References

[1] J.H. Schwarz, *Lectures on superstring and M-theory dualities*, *Nucl. Phys.* **55B** *(Proc. Suppl.)* (1997) 1 [hep-th/9607201].

[2] O. Aharony, J. Sonnenschein and S. Yankielowicz, *Interactions of strings and D-branes from M-theory*, *Nucl. Phys.* **B 474** (1996) 309 [hep-th/9603009].

[3] K. Dasgupta and S. Mukhi, *BPS nature of 3-string junctions*, *Phys. Lett.* **B 423** (1998) 261 [hep-th/9711094].

[4] A. Sen, *String network*, *J. High Energy Phys.* **03** (1998) 005 [hep-th/9711130].

[5] O. Aharony, A. Hanany and B. Kol, *Webs of (p,q) 5-branes, five dimensional field theories and grid diagrams*, *J. High Energy Phys.* **01** (1998) 002 [hep-th/9710116].

[6] A. Mikhailov, N. Nekrasov and S. Sethi, *Geometric realizations of BPS states in $N=2$ theories*, *Nucl. Phys.* **B 531** (1998) 345 [hep-th/9803142].

[7] B. Kol, *Thermal monopoles*, *J. High Energy Phys.* **07** (2000) 026 [hep-th/9812021].

[8] N. Sasakura, *Low-energy propagation modes on string network*, *J. High Energy Phys.* **02** (2001) 033 [hep-th/0012270].

[9] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315].

[10] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B 557** (1999) 79 [hep-th/9810155].

[11] T. Regge, *General relativity without coordinates*, *Nuovo Cim.* **19** (1961) 558.

[12] R. Penrose, in *Quantum theory and beyond*, T. Bastin, Cambridge University Press 1971.

[13] C. Rovelli and L. Smolin, *Spin networks and quantum gravity*, *Phys. Rev.* **D 52** (1995) 5743 [gr-qc/9505006].

[14] H. Ooguri and N. Sasakura, *Discrete and continuum approaches to three-dimensional quantum gravity*, *Mod. Phys. Lett.* **A 6** (1991) 3591 [hep-th/9108006].

[15] J. Ambjørn, B. Durhuus and T. Jonsson, *Quantum geometry. A statistical field theory approach*, Cambridge University Press, 1997 p. 363.

[16] S.-J. Rey and J.-T. Yee, *BPS dynamics of triple (p,q) string junction*, *Nucl. Phys.* **B 526** (1998) 229 [hep-th/9711202].

[17] C.G. Callan Jr. and L. Thorlacius, *Worldsheet dynamics of string junctions*, *Nucl. Phys.* **B 534** (1998) 121 [hep-th/9803097].

[18] P. Shocklee and L. Thorlacius, *Zero-mode dynamics of string webs*, hep-th/0101080.